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Multivariate Optimum Design of a Subsonic Jet Passenger Airplane

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Optimum airplane design plays a vital role in the development of efficient and effective aircraft because it primarily relies upon the best possible combination of excellent aerodynamic qualities, lightweight structure, economy of operation, etc., which satisfy required flight characteristics. Optimum design procedures are intended to reduce the dependence upon a designer's individual skill, experience, and intuition. In the present work, an attempt has been made to use a multivariate optimization technique in the preliminary aerodynamic design of a civil subsonic jet aircraft. The design problem is formulated as a constrained nonlinear optimization problem. The solution is obtained by using the complex method of M.J. Box. The necessary relations used for weight estimation, aerodynamic characteristics, engine properties, etc., are either simplified or empricial in nature. However, the program developed can easily be modified to include more accurate formulas and additional conditions if desired. The numerical exercise carried out here is mainly for illustrating the approach suggested.

Nomenclature

A_w = wing aspect ratio
A_{fn} = ratio of length to characteristic diameter for fuselage or engine
a = speed of sound
b = wing span
C_{lf} = ratio of fuselage diameter to square root of wing planform area
C_L = coefficient of lift of an airplane
C_D = coefficient of drag of an airplane
C_{Do} = coefficient of airplane drag at zero lift
C_{Dfn} = coefficient of drag for fuselage or engine nacelle
C_{Dwp} = coefficient of profile drag for wing
C_{Dt} = coefficient of profile drag for tail
C_p = specific fuel consumption
C_T = coefficient of thrust ($=T/0.5\rho V^2 S$)
D = airplane drag
D_f = fuselage diameter
F_l = objective function
g = gravitational acceleration ($=9.81$ m/s^2)
H = altitude of flight
L = lift acting on airplane
L_p = lapse rate of temperature
L_d = design range
M = cruising Mach number ($=V/a$)
M_{cr} = critical Mach number
n = load factor
p_o = design wing loading ($=W_o/S$)
Re = Reynold's number
S = wing planform area
ΣS_m = summation of the frontal areas of cross section for fuselage and external engines
S_t = planform area for tail plane
TE = temperature, K
$\bar{t}_o$ = thickness ratio of wing
V = cruising speed
W, W_o = weight of airplane at any condition and its design value
W_{pp} = weight of powerplant
W_{str} = weight of fuel
W_p = payload
α = throttle coefficient
η = taper ratio of wing (= root chord/tip chord)
Δ = density ratio of air
ρ, ρ_o = density of air at any altitude and at sea level, respectively
μ = coefficient of dynamic viscosity of air
ψ = coefficient representing Mach number effect on fuel consumption
χ, χ_t = wing sweep at quarter chord line and trailing edge, respectively

Introduction

AN airplane design is a complex process of applying the fundamentals of aerodynamics, structures, powerplants, stability, and control with a certain degree of judgment exercised by an individual designer. Various requirements have to be satisfied during this process, some of which often result in contradictions between the technological and operational objectives or between versatility and specific performance. It is, therefore, necessary to compromise and find the optimum solution for achieving an efficient airplane design.

There are basically two types of problems in airplane design: direct and reverse. The direct problems are those which emerge from the natural design sequence and requirements of customer. In reverse problems, the designer treats some details as known besides the primary requirements of the customer. There are numerous works[1-6] available in the literature which discuss the problem of optimum design analysis and computer-aided airplane design. Thus, the present problem of designing an airplane around a given engine and satisfying the customer's requirements is a reverse design problem.

An attempt has been made here to arrive at the preliminary aerodynamic design by simultaneous optimization of the leading airplane parameters with two separate design criteria:

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maximum fuel efficiency and maximum payload. The main aim of the present work is to develop a computer program for this problem using the feasible direction method of Box[7] involving nonlinear constrained optimization. The software developed here is quite flexible and can easily allow alterations or additions in the objective function as well as in the constraints to be imposed. The numerical solutions of problems presented are for the purpose of illustrating the method indicated and can be improved by incorporating more accurate drag and weight estimations in place of those given in Appendices C and D.

Formulation of Problem

Statement of Problem

It is required to find the best combination of airplane parameters defining the preliminary design such that the desired objective function is optimized to satisfy all of the specified constraints.

In the first case, an objective function is the fuel efficiency defined by the relation[8]

$$FI = \frac{\text{payload} \times \text{range}}{\text{fuel weight}} \tag{1}$$

This objective function obviously combines the design criteria of payload, range, and fuel economy appropriately. This function has to be maximized.

In the second case, the payload (consisting of the weights of passengers, crew, cargo, food, seats, etc.) is taken as the objective function to be maximized for a given powerplant.

Constraints

The constraints imposed during the design process are of two types: explicit and implicit. The upper and lower bounds of all the independent design parameters are specified as predetermined fixed values from necessary practical considerations. They form the explicit constraints. The implicit constraints are specified depending upon the performance requirements and compatibility. For the present problem these constraints are determined from considerations of takeoff and cruise requirements. Typical implicit requirements and the relations used for weight and drag estimation of the airplane are given in Appendices B-D.

Design Parameters

The independent parameters considered here for defining the preliminary design of airpalne and evaluating the objective function are: H, M, A_w $\bar{t}_o$, χ, η, D_f, A_{fn} (fuselage), C_{lf}, and p_o.

Specified Data

The preassigned values and customers' requirements shown in Table 1 are taken as data for solution of this problem.

Method of Optimization

The feasible direction method by Box[7,9] has been used here for solving the problem. The flow diagram for this method is

Table 1 Specified data

Engine	JT-3-D
Maximum sea level static thrust per engine	80,200 N
Maximum sea level static sfc, C_{po}	0.535 N/N·h
Lift coefficient during takeoff, C_{Lto}	1.8
Takeoff ground run, S_G	2000 m
Design range, L_d	6000 km
Number of engines (variable), N_e	2 to 4
Weight of each engine, W_{eng}	18,550 N
Average coefficient of friction between ground and landing gear wheels during takeoff, f_{to}	0.02

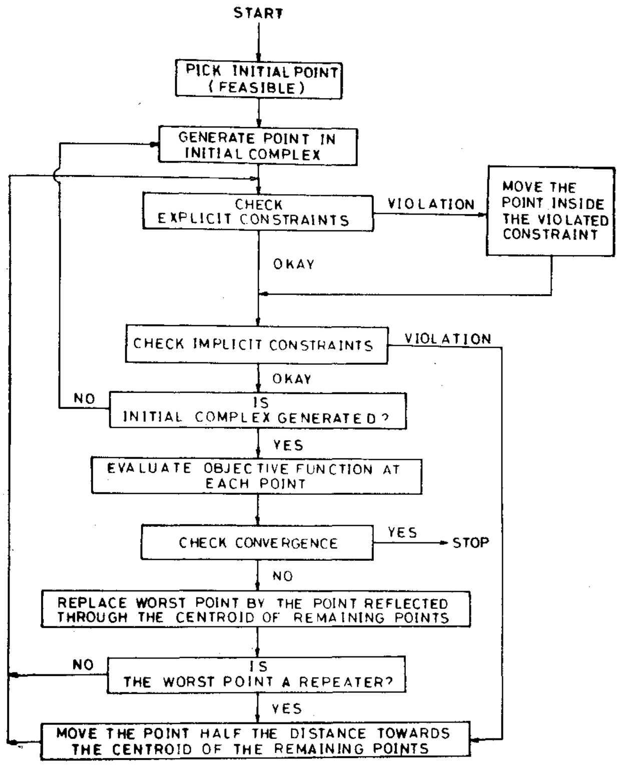

Fig. 1 Flow diagram illustrating Box method.

shown in Fig. 1. The algorithm for the solution of this problem is as follows:

1) The values of the first nine design parameters (i.e., except wing loading) are generated by random number generation as stated below:

$$X_{ij} = G_j + r_{ij}(H_j - G_j) \tag{2}$$

where

X_{ij} = value of jth variable at ith point ($i = 1,2,\ldots,K$)
G_j = lower boundary on jth variable ($j = 1,2,\ldots,N$)
H_j = upper boundary on jth variable ($j = 1,2,\ldots,N$)
r_{ij} = random number between 0 and 1

2) After these values are fixed, the upper boundary on wing loading is decided as indicated in the following steps:

a) The fuselage length L_f is calculated from selected values of D_f and A_{fn} (fuselage).

b) The wing span b is then calculated from the following relation[5]:

$$b = L_f (A_w)^{1/2} / (C_{lf} \cdot A_{fn}) \tag{3}$$

c) The wing planform area is estimated from the definition of wing aspect ratio:

$$S = b^2 / A_w \tag{4}$$

d) The thrust loading (T_o/S) can now be calculated since takeoff thrust T_o is known for the given engine.

e) The maximum allowable wing loading from takeoff consideration for the above relation of thrust loading and takeoff performance (see Appendix B) is calculated.

f) For this value of wing loading and the randomly generated values of other parameters together with known atmospheric properties, the drag and available thrust are calculated (see Appendices A-C for typical relations required) for cruising attitude.

g) If the drag coefficient C_D is less than the thrust coefficient C_T, the same value of p_o determined in step e is taken as an upper boundary on wing loading.

h) If C_D is greater than C_T, then the wing loading is reduced. The drag and thrust available are estimated again. This procedure is continued until C_T becomes greater than C_D. If the basic drag coefficient C_{Do} itself is greater than C_T, then the cruising Mach number is reduced. The wing loading which satisfied the cruise thrust requirement is then chosen as the upper boundary on wing loading.

i) Then the point is chosen in the design space by incorporating random number generation for the last variable p_o.

3) Once the initial complex is complete, the objective function is evaluated at each selected point in the design space after estimating various weight fractions (Appendix D).

a) The objective function for the first case is worked out from the relation: $F1 = W_{pl} \cdot L_d / W_{fu}$

b) Weight of the powerplant W_{pp} is known (given).

c) The weight of fuel is estimated in parts as follows:

i) Weight of fuel in cruise = applied thrust $\times$ $C_p \times L_d$/cruise speed.

ii) Fuel for takeoff, climb, and landing is estimated from an empirical relation (Appendix D).

iii) Reserve fuel is estimated from statistical considerations.

d) The structural weight of the airplane is estimated using the empirical formula given in Appendix D.

e) The first estimate of payload is made from the weight balance equation. Further, the maximum number of passengers that can be accommodated is worked out on the basis of fuselage dimensions and the seating arrangement in passenger cabin. The second estimate of payload is made using the standard statistical norm, i.e., taking 220g newtons as the payload per passenger (see Appendix D). The minimum value of these two estimates is taken as a payload when evaluating the objective function.

4) Selected points must satisfy the explicit and implicit constraints. If an explicit constraint is violated, the point is shifted by a small distance δ inside the violated limit. If the implicit constraint is violated, then the point is moved by half the distance toward the centroid of the rest of the points so that

$$X_{ij\text{new}} = (X_{ij\text{old}} + \bar{X}_{cj})/2 \tag{5}$$

where $\bar{X}_{cj}$ is the centroid of the rest of the points given by

$$\bar{X}_{cj} = \left(\sum_{i=1}^{K} X_{ij} - X_{ij\text{old}}\right)/(K-1) \tag{6}$$

$K \geq N+1$ is the number of points.

This process is repeated until all of the constraints are satisfied.

5) The objective function is evaluated at each point. The point with worst value is replaced by another point located at a distance d from the worst point along the line joining this rejected point and the centroid. Here, d is equal to α_I times the distance from this centroid of the rest of the points and the rejected point. Thus,

$$X_{ij\text{new}} = X_{ij\text{old}} + (1+\alpha_I)(\bar{X}_{cj} - X_{ij\text{old}})$$

6) If a point repeatedly gives the worst value on consecutive trials, it is moved by half the distance toward the centroid of the remaining points.

7) The new point is checked to see whether the constraints are satisfied and is adjusted as above if necessary.

8) This procedure is continued until the desired convergence criterion is satisfied.

In the present problem, the initial number of points chosen is 21 ($2N+1$, $N=10$ is the number of desired variables) as indicated in the method of Box. The reflection factor α_I is taken as 1.3.

Results and Discussion

The computer program has been developed in FORTRAN-IV language and run on the EC-1030 system at the Indian Institute of Technology. Global optima for various cases have been confirmed through the same end results for different initial sets of variables. The final sets of the given parameters defining preliminary designs of aircraft were obtained for different cases and are presented in Table 2. An attempt has been made to modify Box's complex method. The reflections of worst points were taken around the weighted centroid instead of the simple centroid. The weight was decided by the values of the objective function. It was observed that although faster movement toward the centroid was achieved initially, this advantage was not retained after a few tens of

Table 2 Optimized values of various independent parameters

Variants	1	2	3	4	5	6	7
H,km	13.61	12.03	13.34	11.36	14.25	15.81	9.927
M	0.897	0.899	0.899	0.899	0.897	0.899	0.830
A_W	9.00	8.99	9.00 (fixed)	8.99	8.96	8.94	6.07
A_f	10.00	10.00	10.00	10.00	10.44	10.19	11.65
N_e(fixed)	2	2	2	4	4	4	4
$\bar{t}_o$	0.079	0.088	0.088	0.113	0.104	0.101	0.148
χ, deg	22.81	22.97	22.85	23.42	23.07	24.02	
D_f, m	3.00	3.01	3.01	4.80	4.86	4.91	6.28
η	2.09	2.13	2.12	2.16	2.16	2.06	3.29
C_{lf}	0.25	0.25	0.25	0.283	0.295	0.299	0.300
p_o, N/m^2	4570	6560	4565	5565	4110	3360	4360
$\frac{W_{pl} \cdot L_d}{W_{fu}} \times 10^{-3}, \frac{\text{N} \cdot \text{km}}{\text{N}}$	14.51	11.42	12.95	11.44	9.866	9.217	...
$W_{pl} \times 10^{-3}$, N	...	...	...	...	400 ($N_p = 180$)	311 ($N_p = 140$)	622 ($N_p = 280$)
$W_o \times 10^{-3}$, N	67.3	82.2	67.2	164.0	113.5	92.2	192
L_d, km	5000 (fixed)	5000 (fixed)	5928	6000 (fixed)	6000 (fixed)	6000 (fixed)	6000 (fixed)
C_{Lto}(fixed)	1.5	1.8	1.8	1.8	1.8	1.8	1.8
Remarks[a]	NR	NR	NR	NR	R	R	R

[a] NR: No restriction on payload imposed. R: Payload restricted or given.

iterations. It should be noted that the payload in the present problem has been empirically calculated and various weight components such as seats, food and buffet equipment, air conditioning system, etc., have been included in the same payload.

It can be seen that the system developed in this work, with fuel efficiency as the evaluation criterion, gives an optimum design having higher wing aspect ratio, high cruise altitude and Mach number, somewhat low sweep, lower thickness ratio, and lower taper ratio of wing. The stress has been mainly on fuel saving as it appears in the numerator as well as in the denominator of the objective function. This has been attempted even at the cost of a small structural weight penalty caused by the high aspect ratio and lower thickness and taper ratios of the wing. The low sweep compensates for this structural weight penalty to some extent. The high cruise altitude and low thickness ratio account of comparatively small values of sweep. The wing loading also seems to be somewhat lower to decrease the structural weight penalty.

All of these intricate adjustments are due to the objective function, as can be seen from the results obtained for different objective functions. When maximizing the payload is taken as the design criterion, the emphasis seems to be stressed equally upon reduction in both structural and fuel weights. In this case, the cruising altitude, cruising Mach number, and wing aspect ratio show lower values; whereas the wing loading, wing taper and thickness ratios, fuselage fineness ratio, and fuselage diameter are higher. The software system developed here seems to be sensitive to the objective functions and design requirements.

Appendix A: Atmospheric Properties

$TE = TE_o - L_p H; L_p = 6.5\times10^{-3}$, K/m for $H \le 11{,}000$ m

$TE = TE_o - 71.5$ for $H > 11{,}000$ m

$\rho = \rho_o (1 - H/44{,}300)^{4.256}$ for $H \le 11{,}000$ m

$\rho = \rho_{11,000} \cdot \exp(11{,}000 - H/6340)$ for $H > 11{,}000$ m

$\mu_{TE} = \mu_o (TE/TE_o)\ 0.75$

$a = 20.066\,(TE)^{0.5}$

$M = V/a$

$\Delta = \rho/\rho_o$

Note: Suffix o indicates sea level condition.

Appendix B: Engine and Airplane Performance

Engine Characteristics[10]

$\alpha = T_{\text{applied}} / T_{\text{available}}$

$T = \alpha \cdot T_o \cdot (\Delta)^{0.85}$ for $H \le 11{,}000$ m

$T = 1.2\,\alpha \cdot T_o \cdot \Delta$ for $H > 11{,}000$ m

$C_p = C_{po} \cdot \psi \cdot \Delta^{0.12}[0.96 + (\alpha - 0.8)^2]$ for $H \le 11{,}000$ m

$C_p = 0.863\, C_{po} \cdot \psi [0.96 + (\alpha - 0.8)^2]$ for $H > 11{,}000$ m

$\psi = 1.05 + 0.1M + 0.05\,M^2 \quad M \ge 0.8$

$\psi = 1.0 \quad M < 0.8$

Level flight relations:

$$W = L = \tfrac{1}{2}\rho V^2 S C_L; T = D = \tfrac{1}{2}\rho V^2 S C_D$$

Takeoff Performance[11]

$$(T_o/S)_{\text{required}} = \frac{p_0^2}{0.9 C_{Lto} \cdot \rho \cdot g \cdot S_G} + (1.1 f_{to} + 0.033) p_o$$

Appendix C: Drag Polar and Drag Estimation[11]

Drag polar relation: $C_D = C_{Do} + k \cdot C_L^2$

$C_{Do} = C_{Dwp}(1 + C_{Dt} \cdot S_t / C_{Dw} \cdot S) + p_o \Sigma (C_{Dfn} \cdot S_m / W_o)$

$C_{Dwp} = 2C_{fp}(1 + 3\bar{t}_o)(1 + \bar{t}_o M)\bar{S}_{\text{wet}} + 0.001 L'$

$\bar{S}_{\text{wet}} = 1 - k_{\text{int}} \cdot \bar{S}'$; $k_{\text{int}} = 0.9$, 0.7, and 0.5 for high wing, middle wing, and low wing, respectively.

$\bar{S}' = (D_f/b)[2 - D_f/b(1 - 1/\eta)]/(1 + 1/\eta)$

$\bar{L}' = L'/b$ = ratio of the spanwise length occupied by the ailerons and flap to total wing span.

$$C_{fp} = \frac{C_f'}{(1 + 0.1M^2)^{2/3}}\left[1 - \bar{x}_{\text{trans}} + \frac{40}{R_e^{3/8}}(\bar{x}_{\text{trans}})^{5/8}\right]^{4/5}$$

$C_f' = 0.458/\log_e Re)^{2.58}$

$$C_{Dfn} = 0.0075 A_{fn}\left(1 + \frac{2.3}{A_{fn}^{1.4}}\right)\left[1 + 0.65M^6\left(\frac{2.72}{A_{fn}^{1/3}} - 1\right)\right]$$

$$k = 1/\pi A_{we}; A_{we\,\text{incomp}} = \frac{A_w}{1 + r_{\text{incomp}}};$$

$$A_{we\,\text{comp}} = \frac{A_{we\,\text{incomp}}}{1 + r_{\text{comp}}}$$

$$r_{\text{incomp}} = \frac{0.02 A_w}{\cos\chi}\left(3.1 - \frac{14}{\eta} + \frac{20}{\eta^2} - \frac{8}{\eta^3}\right)$$

$r_{\text{comp}} = 0; M < M_{cr}''$

$= A_w(\bar{t}_o)^{1/3}(M - M_{cr}'')^3; M > M_{cr}''$

$M_{cr}'' = M_{cr}' - C^{3/2}{}_L(\bar{t})^{1/2}; C_L > 0$

$$M_{cr}' = \frac{1}{\cos\chi_t}\left[1 + \frac{(\gamma + 1)^{4/3}}{2} \cdot \frac{(\bar{t})^{4/3}}{(\cos\chi_t)^{2/3}} - \frac{(\gamma + 1)^{2/3}(\bar{t})^{2/3}}{(\cos\chi_t)^{1/3}}\right]$$

γ = ratio of specific heat at constant pressure to that at constant volume.

Appendix D: Weight Estimation of Airplane[4,10,11]

Weight balance equation:

$$\bar{W}_{str} + \bar{W}_{fu} + \bar{W}_{pp} + \bar{W}_{pl} + \bar{W}_{fix} = 1$$

Structural weight of an airplane with large and medium aspect ratio wings:

$$\bar{W}_{str} = \left[0.027\phi \cdot n \cdot \frac{W_o^{1/2}}{\cos\chi}(A_w/p_o)^{1/2} + 5.5/p_o\right](1 + B_1 A_f m_w + B_2) + 0.065$$

where W_o is in tons and p_o is in kg/m^2

$$\phi = 1 - \frac{2(n+1)}{n+2}(\bar{z}_1 \cdot e_1 \cdot \bar{W}_{fu} + \bar{z}_2 \cdot e_2 \cdot \bar{W}_{eng})$$

B_1 = 0.08-0.11 for high subsonic transport airplanes

m_w = 1.2-1.3 for subsonic airplanes

B_2 = 0.145-0.155 for subsonic and high subsonic airplanes

Fuel Weight

$$\bar{W}_{fuclimb} = \frac{C_{pav}}{1800}\left(\frac{V}{19.6}+\frac{H}{V}\right)$$

$$\bar{W}_{fucruise} = T_{applied}\cdot C_p \cdot \text{range (km)/speed(km/h)}$$

$$\bar{W}_{futotal} = T_o \cdot C_p(\text{range}+0.5\text{ speed})/\text{speed} + 2.5\,\bar{W}_{fuclimb}\cdot W_o$$

$$\bar{W}_{fu} = W_{fu}/W_o$$

Payload

The weight per passenger (in newtons):

Average weight of a passenger	$=75\times g$
Baggage (freely allowable)	$=20\times g$
Cargo weight (including mail)	$=25\times g$
Weight of seat	$=25\times g$
Weight of food, baggage racks, lavatories, buffet equipment, water and air conditioning equipment	$=75\times g$
Total weight per passenger	$=220\times g$
Average weight per crew member	$=100\times g$

Fixed weight $\bar{W}_{fix}=0.04$-0.05 accounts for hydraulic and electrical systems, instruments, surface controls, etc.

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